ENERGY PRINCIPLE

$$\text{The term enthalpy} = h = \left(u + \frac{p}{\rho}\right)$$

$$\dot{Q} - \dot{W}_s = \int_{A2}\left(\frac{p_2}{\rho} + u_2 + \frac{V_2^{\,2}}{2} + gz_2\right)\rho V_2 dA_1 - \int_{A1}\left(\frac{p_1}{\rho} + u_1 + \frac{V_1^{\,2}}{2} + gz_1\right)\rho V_1 dA_1$$

$$\dot{Q} - \dot{W}_s + \int_{A1}\left(\frac{p_1}{\rho} + gz_1 + u\right)\rho V_1 dA_1 + \int_{A1}\frac{\rho V_1^{\,3}}{2}dA_1 = \int_{A2}\left(\frac{p_2}{\rho} + gz_2 + u_2\right)\rho V_2 dA_1 + \int_{A2}\frac{\rho V_2^{\,3}}{2}dA_2$$

It is common to express the term $\int \frac{\rho V^3}{2}dA = \alpha\left(\frac{\overline{V}^3}{2}\right)A$

Substituting for $\int \frac{\rho V^3}{2}dA = \alpha\left(\frac{\overline{V}^3}{2}\right)A$ **in Eqn. above and dividing through by** $\dot{m}$

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1\frac{\overline{V}_1^{\,2}}{2}\right) = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2\frac{\overline{V}_2^{\,2}}{2}\right)$$

Example (7.4)

The pipe in Fig. 7.2 is 50 cm in diameter and carries water at a rate of 0.5 m^3/s. Also, $z_2 = 40$ m, $z_1 = 30$ m, and $p_1 = 70$ kPa gage. What power in kilowatts and in horsepower must be supplied to the flow by the pump if the gage pressure at section 2 is to be 350 kPa? Assume $h_L = 3$ m of water and $\alpha_1 = \alpha_2 = 1$.

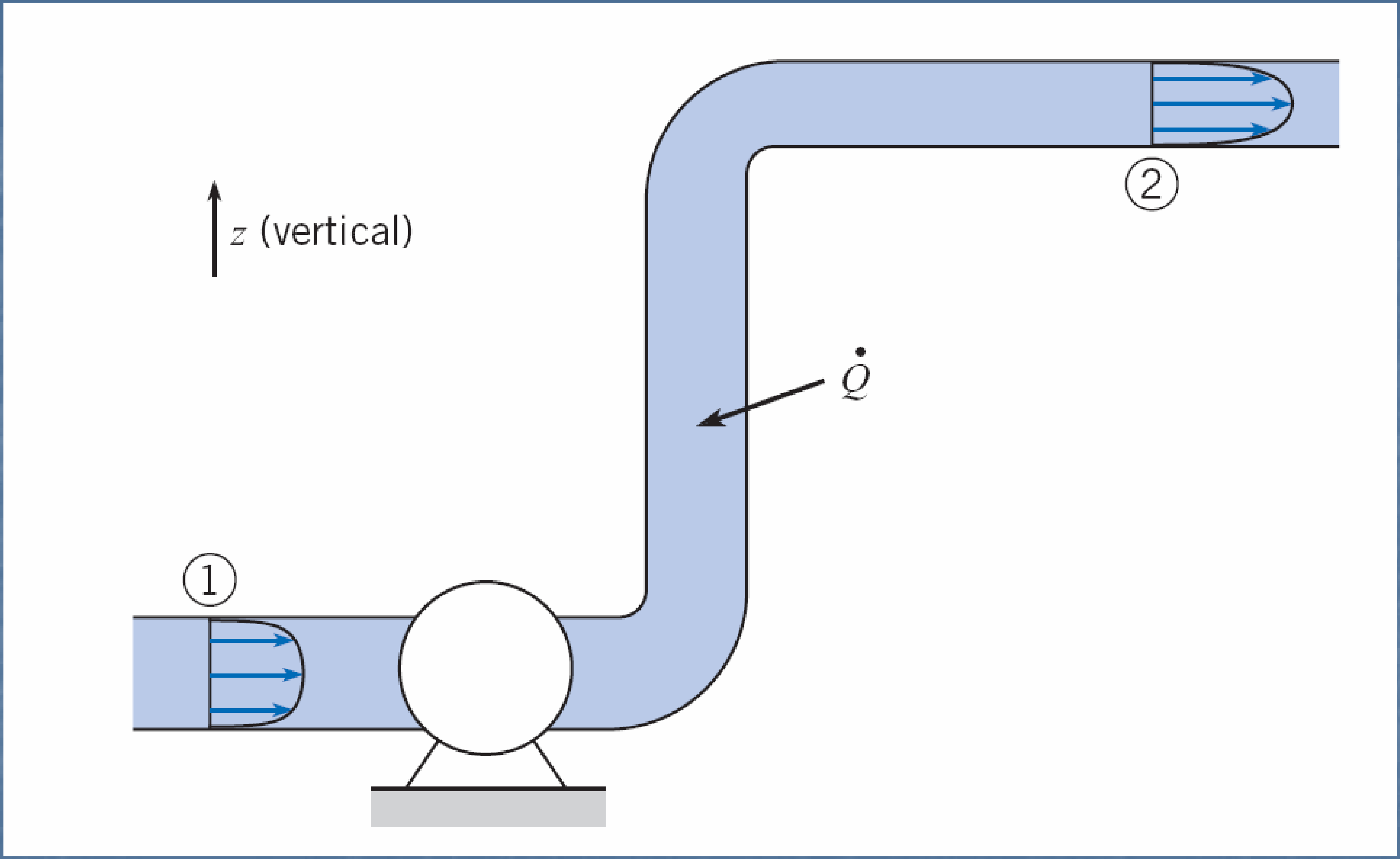
$$V_1 = 0, \quad p_1 = 70\,kPa, \; p_2 = 350\,kPa, \quad z_1 = 30m, \quad z_2 = 40m, \quad V_2 = \frac{\dot{Q}}{A} = \frac{0.05}{\pi(0.05^2/4)} = 2.55\,m/s,$$

$$\alpha_1 = \alpha_2 = 1, \quad h_T = 0, \quad D = 50\,cm, \quad h_{loss} = 3\,m$$

<u>Find:</u> **what is the power of the pump in kW and horse power?**

<u>Applying the energy equation between point 1 & 2</u>

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^2}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\overline{V}_2^2}{2g}\right)_{Mech.part} + h_{Loss}$$



z (vertical)
$\dot{Q}$
1
2

Example (7.4)

$$h_p = \frac{p_2 - p_1}{\gamma} + \frac{V_2^2 - V_1^2}{2g} + z_2 - z_1 + h_L$$

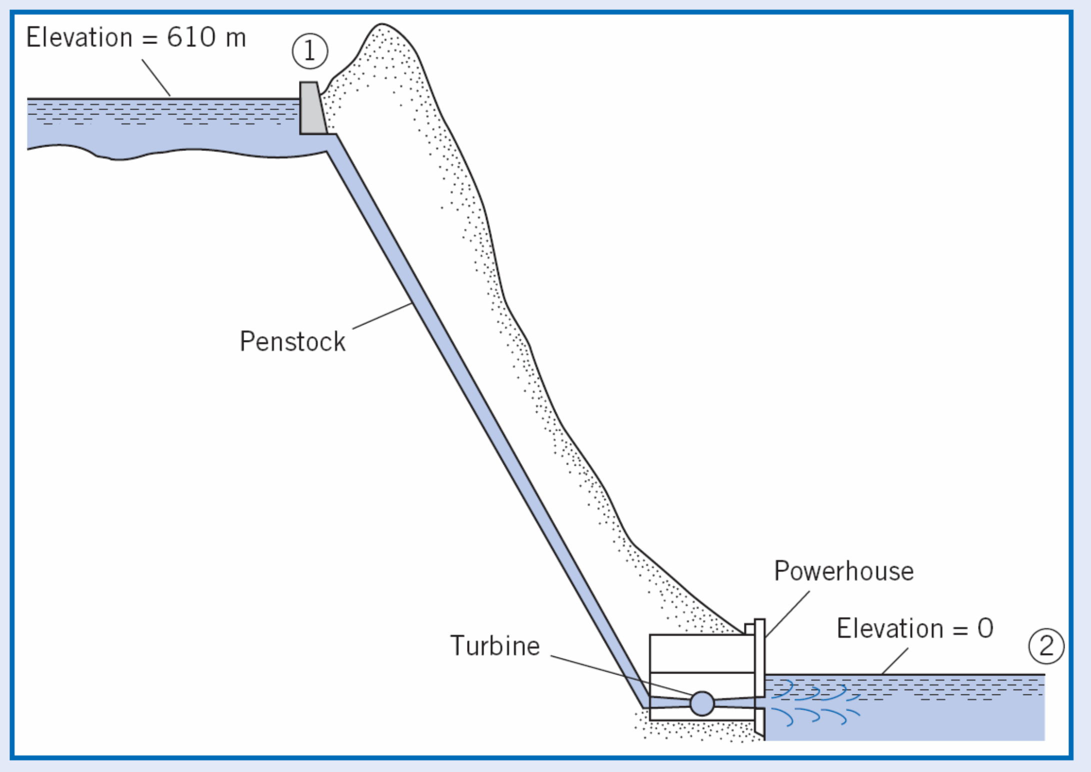
$$= \frac{(350 - 70)\ \text{kN/m}^2}{9.81\ \text{kN/m}^3} + \frac{(2.55^2 - 2.55^2)\ \text{m}^2/\text{s}^2}{9.81\ \text{m/s}^2} + 40\ \text{m} - 30\ \text{m} + 3\ \text{m}$$

$$= 28.5 + 0 + 10 + 3 = 41.5\ \text{m}$$

$$The\ power\ of\ the\ pump = \dot{W}_p = \dot{m}gh_p = \gamma Q h_p = 9.81 \times 0.5 \times 41.5 = 204\ kW = \frac{204}{0.747} = 273\ hp$$

Example (7.5)

At the maximum rate of power generation this hydroelectric power plant takes a discharge of 141 m^3/s. If the head loss through the intakes, penstock, and outlet works is 1.52 m, what is the rate of power generation?



$$Given: \dot{Q} = 141 m^3/s, \quad h_{loss} = 1.52\, m, \quad \alpha_1 = \alpha_2 = 1$$

$$Find: \quad Power\ of\ the\ turbine\ \dot{W}_T\ ?$$

ENERGY PRINCIPLE

Applying the energy equation between point 1 & 2

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + z_2 + \alpha_2 \frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part} + h_{Loss}$$

$$V_1 = 0, \quad p_1 = 0, z_1 = 610m, \quad z_2 = 0, \quad V_2 = 0, \quad \alpha_1 = \alpha_2 = 1, \quad h_p = 0, \quad p_1 = p_2 = 0$$

$$h_t = 610 - 1.52 = 608.5 \text{ m}$$

$$\dot{W}_p = \gamma \dot{Q} h_T = 9.81 \times 141 \times 608.5 = 842\,MW$$

END OF LECTURE (4)